ELECTRICAL MEASUREMENTS & INSTRUMENTATION

UNIT I

Electrical Measurements: Measurement system, Characteristics of instruments, Methods of measurement, Errors in Measurement & Measurement standards, Review of indicating and integrating instruments: Voltmeter, Ammeter and Wattmeter.

1.1.1 MEASUREMENT

Measurement is the act, or the result of a quantitative comparison between a predetermined standard and an unknown magnitude.

1.1.2 MEASURAND

The physical quantity or the characteristics condition which is the object of measurement in an instrumentation system is termed as "measurand".

They may be:

- Fundamental quantity, *e.g.* length, mass and time etc.
- Derived quantity, *e.g.* speed, velocity, pressure etc.

The fundamental measuring process is shown in fig.1.

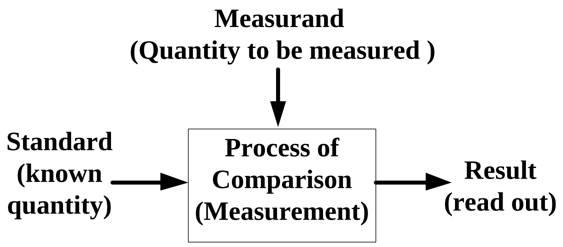

Fig.1. Measurement Process

1.1.3 INSTRUMENTATION

The process/technology of using instruments to measure and control the physical and chemical properties of any material is called instrumentation.

1.2 METHODS OF MEASUREMENT

There are two basic methods of measurement:

1. **Direct Comparison Method:** In this method, the parameters to be measured are directly compared with either a primary or secondary standard.

   Generally, this method is not always the most accurate or the best method, it is not sensitive enough too.

2. **Indirect Comparison Method:** In this method, the comparison is done with a standard through the use of a calibrated system.

   A measurement system may consist of transducing elements which convert the quantity to be measured with an analogous signal. This signal after being processed by some intermediate means then fed to the end devices which provide the measurement results.

1.3 MODES OF MEASUREMENT

Following are the three basic modes of measurement:

1. **Primary Measurement:** In this case, the desired value of parameter is determined by comparing it directly with "reference standard".

   There is no conversion of measurand.

   e.g. measurement of length using meter scale.

2. **Secondary Measurement:** In secondary measurement system, "one translation" is involved during the measurement process.

   e.g. pressure measurement by manometer.

3. **Tertiary Measurement:** The indirect measurement involving "two conversions" are called tertiary measurement.

   e.g. thermocouple.

1.4 MEASURING INSTRUMENTS

An instrument is a device in which we can determine the magnitude or value of the quantity to be measured. The measuring quantity can be voltage, current, power and energy etc.

Generally instruments are classified in to two categories.

A. Absolute instrument
B. Secondary Instrument

A. Absolute instrument

An absolute instrument determines the magnitude of the quantity to be measured in terms of the instrument parameter. This instrument is really used, because each time the value of the measuring quantities varies. So we have to calculate the magnitude of the measuring quantity, analytically which is time consuming. These types of instruments are suitable for laboratory use.

e.g. Tangent galvanometer.

B. Secondary instrument

This instrument determines the value of the quantity to be measured directly. Generally these instruments are calibrated by comparing with another standard secondary instrument.

Examples of such instruments are voltmeter, ammeter and wattmeter etc. Practically secondary instruments are suitable for measurement.

These instruments can be further divided as:

1. Indicating Instruments;
2. Recording Instruments &
3. Integrating Instruments.

1. Indicating Instruments:

- ➢ Indicating instruments are those which indicates the instantaneous value of the electrical quantity being measured at the time which it is being measured.
- ➢ Their indication is given by pointers, moving on a calibrated scale.

2. Recording instrument:

- ➢ This type of instruments records the magnitude of the quantity to be measured continuously over a

specified period of time.
- The pointer of such instruments leaves a trace on a paper by using a pen.

3. **Integrating Instruments:** This type of instrument gives the total amount of the quantity to be measured over a specified period of time. *e.g.* Amp-hr meter, etc.

1.5 ESSENTIAL FEATURES OF INDICATING INSTRUMENTS

For satisfactory operation electromechanical indicating instrument, should have three essential systems, they are:

1. Deflecting System;
2. Controlling System &
3. Damping System.

1. Deflecting System

- When there is no input signal to the instrument, the pointer will be at its zero position. To deflect the pointer from its zero position, a force is necessary which is known as deflecting force. ***A system which produces the deflecting force is known as a deflecting system***. Generally a deflecting system converts an electrical signal to a mechanical force.

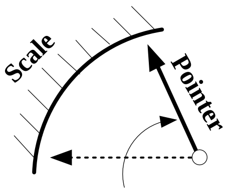

Fig.2. Pointer Scale

- There are few effects which are responsible for the production of deflecting torque as:
  - magnetic effect;
  - heating effect;
  - electrostatic or electromagnetic effect, etc.

2. Controlling System

- To make the measurement indicated by the pointer definite (constant) a force is necessary which will be acting in the opposite direction to the deflecting force. This force is known as controlling force. ***A system which produces this force is known as a controlled system.***
- When the external signal to be measured by the instrument is removed, the pointer should return back to the zero position. This is possibly due to the controlling force and the pointer will be indicating a steady value when the deflecting torque is equal to controlling torque.
- Under steady state of pointer: $T_c = T_d$.

There are two types of controlling devices:

a. Spring Control &
b. Gravity Control.

a. Spring Control: In this system,

- Two springs are attached on either end of spindle.
- Both the springs are wound in opposite direction so that when the moving system is deflected, one spring windup while the other unwinds and the controlling torque produce due to the combined torsion of the springs.
- When a current is supply, the pointer deflects due to rotation of the spindle.
- While spindle is rotate, the spring attached with the spindle will oppose the movements of the pointer.
- The torque produced by the spring is directly proportional to the pointer deflection "θ",

  i.e. $T_c \propto \theta$.
- The deflecting torque produced T_d proportional to current passes through instrument, *i.e.* $T_d \propto I$.
- When pointer will come to a steady position: $T_c = T_d$.

  therefore, $\boxed{\theta \propto I}$

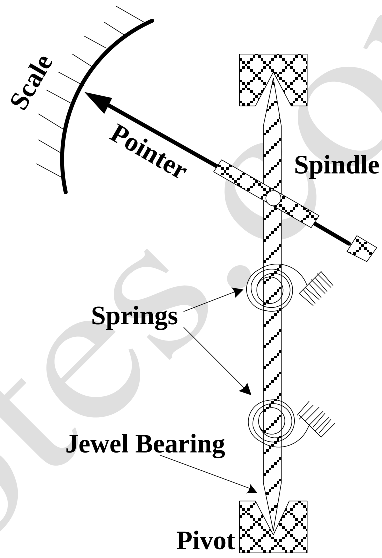

Fig.3. Spring Control

- Since, θ and I are directly proportional; the scale of such instrument which uses spring control is uniform.

The material of spring used should have the following properties:

i. It should be non-magnetic.
ii. Its temperature co-efficient should be low.
iii. Its specific resistance should be low, etc.

b. Gravity Control:

- In gravity control, two weights L and M are attached to the spindle 'S' (as shown in fig.4).
- Weight L is used to balance the weight of the pointer and weight M provide the controlling torque.

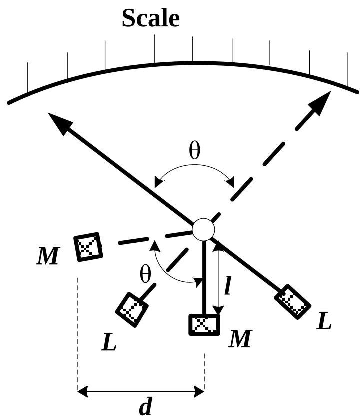

Fig.4. Gravity Control

- During the zero position of pointer, *M* hangs vertically downwards, but when pointer is deflected through an angle θ, the controlling torque will: $T_c = M \times d = M \times l \sin\theta$, i.e. $\boldsymbol{T_c \propto \sin\theta}$.
- Above relation shows that, the relation between deflection and is nonlinear; hence the scale of the instrument is non uniform.

Advantages:

i. These instruments are cheaper as compare to spring controlled instruments.
ii. There is no effect of temperature.
iii. They are not subjected to fatigue.

Disadvantages:

i. In these instruments, proper leveling is required before use.
ii. They give cramped scale.

3. Damping System

When a system is subjected to the deflecting and controlling torque due to inertia of the moving system, the pointer oscillates about its final steady position before coming to rest. To avoid this, a damping torque is required which opposes the oscillations. Such system is called "*damping system*".

This damping force is provided by different systems on behalf of which damping systems are classified as:

a. Air Friction Damping
b. Fluid Friction Damping
c. Eddy Current Damping

a. Air Friction Damping: In such systems

- The piston is mechanically connected to spindle through the connecting rod (as shown in fig.5).

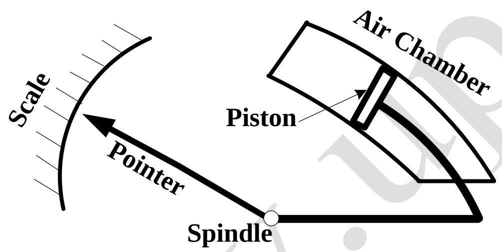

Fig.5. Air Friction Damping

- This piston moves in a fixed chamber (either it circular or rectangular) which is closed at one end.
- When the pointer oscillates in clockwise direction, the piston goes inside and the cylinder gets compressed. The air pushes the piston upwards and the pointer tends to move in anticlockwise direction.
- The vice-versa action is performed when piston oscillates in anticlockwise direction.
- With this damping system care must be taken to ensure that the arm carrying the piston should not touch the sides of the chamber during its movement. The friction which otherwise would occur may introduce a serious error in the deflection.
- Such damping arrangement is very simple and cheap and generally used in moving iron and hot wire instruments.
- There must be take care about the fact that the piston is not bent or twisted.

b. Fluid Friction Damping:

- Such damping system is similar to air friction damping and its action is also similar to the air friction damping.
- In these systems, generally mineral oil is used in place of air and as the viscosity of oil is greater, the damping force is also much greater.
- In these damping systems, a vane attached to the spindle is arranged to move in the damping oil.
- Fig.6 (a) shows the basic arrangement of a fluid friction damping system in which a disc attached to the moving system is immersed in the fluid (used as damping substance).

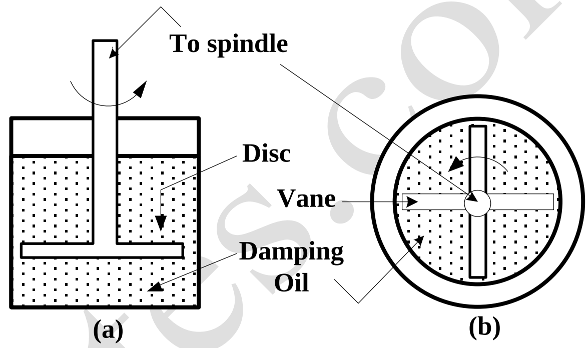

Fig.6. Fluid Friction Damping

- When the moving system moves the disc moves in fluid and a frictional drag is produced.
- For minimizing the surface tension affect, the suspension stem of the disc should be cylindrical and of small diameter.
- For greater damping torque, a number of vanes are attached to the spindle as shown in fig.6 (b).
- These vanes are submerged in fluid and moves in a vertical plane.
- This system is rarely used in commercial type instruments.

The fluid used in these damping system, must fulfill the following requirements:

- The evaporation rate of must be very low.
- It should not have any corrosive effect on metals.
- The viscosity of the fluid should not change appreciably with temperature.
- It should be good insulator.

Advantages

- The fluid (oil) used for damping can also be used for insulation purpose in some forms of instruments which are submerged in oil.
- The clearance between the vanes and oil chamber is not as critical as with the air friction damping system.
- This method is suitable for use with instruments such as electrostatic type where the movement is suspended rather than pivoted.
- Due to the up thrust of oil, the loads on bearings or suspension system is reduced thereby reducing the frictional errors.

Disadvantages

- The instruments with this type of damping must be kept always in a vertical position.

- It is difficult to keep the instrument clean due to leakage of oil.
- It is not suitable for portable instruments.

The fluid friction damping can be used for laboratory type electrostatic instruments.

c. Eddy Current Damping:

- The basic principle of this damping is- ***when a conducting nonmagnetic material is moved in a magnetic field, an e.m.f. is induced in it which causes current, called eddy currents.***
- Due to these currents a force exists between the material and the field.
- By Lenz's law, this force is always in opposition to the force causing rotation of the conducting material; thus it produces the necessary damping.

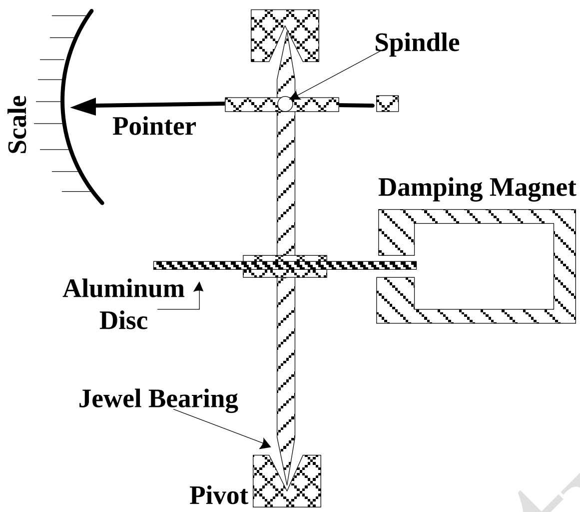

Fig.7. Eddy Current Damping (Disc Type)

1.6 PERMANENT MAGNET MOVING COIL INSTRUMENT

Permanent magnet moving coil (PMMC) instrument is one of the most accurate instruments for d.c. measurement.

Construction: In such instruments,

- A permanent magnet is used to develop a magnetic field.
- A rectangular coil which consists of insulated copper wire wound on light aluminum frame, placed between the two poles of magnet.
- The aluminum frame is connected with the spindle and this spindle is supported with jeweled bearing.
- To provide controlling torque, spring control is used in which two springs are attached on either end of the spindle.
- In such instruments, eddy current damping is used. This is produced by aluminum frame.

Principle of Operation:

- A PMMC type instrument works on the principle that, "***when a current-carrying conductor is placed in a magnetic field, it experience a force on it***".
- This force produces a torque and the frame rotates. Due to which pointer moves over the calibrated scale.
- If the polarity is reversed a torque is produced in the opposite direction. But there are mechanical stoppers to stop the deflection in opposite direction.
- If a.c. is supplied, a fluctuating flux is produced which cannot produce a continuous deflection.

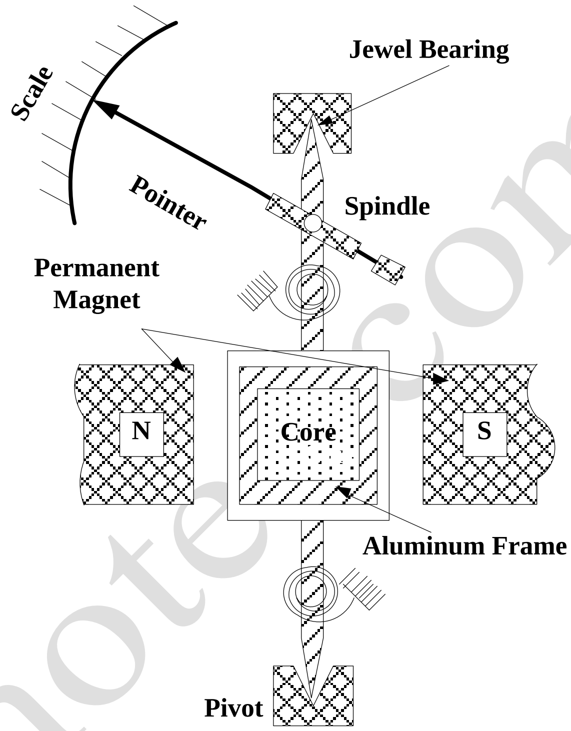

Fig.8. PMMC Instrument

Torque developed by PMMC Instrument:

Let, l=height of the coil or length of coil;
I = current passing through coil;
B = Flux density;
N = No. of turns,
b = width of the coil

Then, force produced in the coil is given by:

$$F = NBIl \quad \text{Newton} \tag{6.1}$$

Deflecting torque produced,

$$T_d = F\times\text{perpendicular distance}$$

i.e. $T_d = F\times b = NBI(l\times b)$

or $$\boxed{T_d = NBIA} \tag{6.2}$$

where, A= area of the coil = $(l\times b)$

From eqn.(2), if B remains constant

Then, $$T_d \propto I \tag{6.3}$$

Since such instruments are spring controlled, therefore; $$T_c \propto \theta \tag{6.4}$$

Under steady position of pointer, $T_c = T_d$, hence from eqn.(1.8.3) and eqn.(1.8.4);

we get $$\boxed{\theta \propto I} \tag{6.5}$$

From above equation, it is clear that the scale of such instrument is uniform.

Advantages:

- Torque/weight ratio is high.
- Less power consumption.
- Scale is uniform.
- Range of instrument can be extended.

Disadvantages:

- D.C. use only.
- Frictional and temperature error are present.

1.7 EXTENSION OF RANGE

(1) DC ammeter:

To extend the range of an ammeter, low resistance should be connected in parallel to it, called "***shunt***".

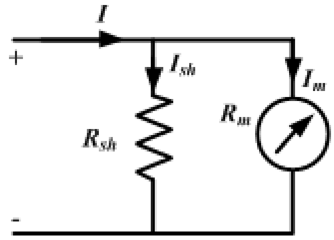

Fig.9(a) Basic D.C. ammeter circuit

Let,

R_m : Internal resistance of the meter,
R_{sh} : Resistance of the shunt,
I_m : Current through the meter,
I_{sh} : Shunt current,
I : Current of the circuit to be measured.

From KCL, $I = I_{sh} + I_m$

or, $I_{sh} = I - I_m$ (7.1)

Since, $V_m = V_{sh}$

or, $I_m R_m = I_{sh} R_m$

or, $$\boldsymbol{R_{sh} = \frac{I_m R_m}{I_{sh}} = \frac{I_m R_m}{I - I_m}} \quad (7.2)$$

Now from, $I = I_m + I_{sh}$

$$I = I_m\left(1 + \frac{I_{sh}}{I_m}\right)$$

or, $$\boldsymbol{I = I_m\left(1 + \frac{R_m}{R_{sh}}\right)} \quad (7.3)$$

$\boldsymbol{\left(1 + \frac{R_m}{R_{sh}}\right)}$ is called multiplication factor.

(2) DC Voltmeter:

A large resistance is connected in series with voltmeter is called "***multiplier***". A large voltage can be measured using a voltmeter of small rating with a multiplier.

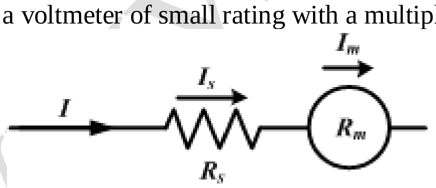

Fig.9(b). Basic D.C. voltmeter circuit

Let,

R_m : Internal resistance of the meter,
R_s : Resistance of the multiplier,
V_m : Voltage across meter,
I_m : Current through meter = (I_s : supply current),
V_s : Voltage across multiplier,
V : Voltage of the circuit to be measured.

Then from KVL, $V = V_m + V_s = I_m(R_m + R_s)$ (7.4)

$\Rightarrow$ $$R_s = \frac{V - V_m}{I_m}$$

or $$\boxed{\boldsymbol{R_s = \frac{V}{I_m} - R_m}} \quad (7.5)$$

Now from, $V = V_m + V_s$

or $$V = V_m\left(1 + \frac{V_s}{V_m}\right) = V_m\left(1 + \frac{I_s.R_s}{I_m.R_m}\right)$$

or $$\boxed{\boldsymbol{V = V_m\left(1 + \frac{R_s}{R_m}\right)}} \quad (7.6)$$

Here, $\boldsymbol{\left(1 + \frac{R_s}{R_m}\right)}$ is called multiplication factor.

1.8 CHARACTERISTICS OF INSTRUMENTS

The quantitative qualities which are use to describe the performance of an instrument are termed as "***characteristics of instrument***". It relates to the degree of approach to perfection.

They are classified as:

1. Static Characteristics,
2. Dynamic characteristics.

1. STATIC CHARACTERISTICS:

Measurements of applications in which the parameters are to be measured, are constant or vary very slowly with time are called "***static characteristics***''.

The classification of static characteristics is:

I. Accuracy: Accuracy of a measuring system is defined as the closeness of the instrument output to the true value of the measured quantity (as per standard).

The accuracy of an instrument can be specified as:

(i) Percentage value of true value
(ii) Percentage value of full scale deflection
(iii) Point accuracy

II. Precision: It is defined as the ability of the instrument to reproduce a certain set of reading within a given accuracy.

III. Sensitivity: The ratio of the magnitude of the change in output signal to the magnitude of change in the input signal of the measuring system to the quantity to be measured is called "***sensitivity***".

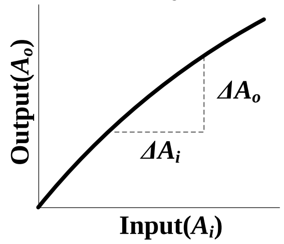

Fig.10. Static Sensitivity

The reciprocal of sensitivity is termed as "***deflection factor***".

Hence, Deflection factor $= \dfrac{1}{sensitivity} = \dfrac{\Delta A_i}{\Delta A_o}$

IV. Resolution: The smallest increment in the measured value that can be detected with certainty by instrument is called "***resolution***".

V. Error: The algebraic difference between the indicated values of the quantity to be measured is called an error.

Mathematically, $e_s = A_m - A_t$ (8.1)

e_s = static error or absolute error,

A_m = Measured value of the quantity,

A_t = True value of the quantity.

Error may be defined as "*relative error* (e_r)",

where; $e_r = \dfrac{Absolute\text{-}error}{True\text{-}value}$

$$\Rightarrow \quad e_r = \frac{e_s}{A_t} = \frac{A_m - A_t}{A_t}$$

Or, $e_r = \dfrac{A_m}{A_t} - 1 \Rightarrow A_t = \dfrac{A_m}{1+e_r}$

Or, $A_t = A_m(1+e_r)^{-1}$ (10.2)

Since the value of error is very small as, $e_r \ll 1$;

So from expansion, the above eqn.(3) may be rewrite as,

$$\boxed{A_t = A_m(1-e_r)} \quad (10.3)$$

Static Correction: $C_s = V_t - V_m = -e_s$.

VI. Linearity: It is a closeness to which a curve approximates a straight line. Linearity is the ability to reproduce the input characteristics symmetry.

VII. Hysteresis: It is the phenomenon where by a change in an effect lack behind change in its cause. The maximum difference in output at any measured value within the specified range when approaching the point first with increasing the input and second with decreasing the output.

VIII. Dead Zone: The maximum change in input for which output is zero is called dead zone.

IX. Drift: It is an undesirable change in input output relationship for a period of time.

2. DYNAMIC CHARECTERISTICS: Dynamics characteristics are those which rapidly changes with time.

1.9 ERRORS IN MEARUREMENTS

Every instrument is specified by within the certain range of accuracy by the manufacturer upto which the measured value may deviates from true value of quantity to be measured.

1.9.1 Limiting Error: "***The limits of deviations from the specified value are defined as limiting error, also called guaranteed error***".

Mathematically the limiting error can be expressed as:

$$\boxed{A_a = A_s \pm \delta A} \quad (9.1)$$

Where; A_a = Actual value of quantity,

A_s = Specified or rated value,

δA = limiting error.

1.9.2 Relative Limiting Error: It is the ratio of the error to the specified magnitude of a quantity (also called fractional error).

i.e. $$e = \frac{\delta A}{A_s} \quad (9.2)$$

From above equation; $\delta A = e.A_s$.

So with the help of above equation and eqn.(11.2), we get;

$$\boxed{A_a = A_s(1 \pm e)} \quad (9.3)$$

1.9.3 COMBINATION OF QUANTITIES WITH LIMITING ERRORS

i. Sum of the Quantities:

Let, q_1 and q_2 be the two quantities which are to be added to obtain the resultant as Q.

Then, $Q = q_1 + q_2$

Now, limiting error in resultant may be written as: $\dfrac{dQ}{Q}$

or, $$\frac{dQ}{Q} = \frac{d(q_1+q_2)}{Q} = \frac{dq_1}{Q} + \frac{dq_2}{Q}$$

$$= \frac{q_1}{Q}.\frac{dq_1}{q_1} + \frac{q_2}{Q}.\frac{dq_2}{q_2}$$

$$\Rightarrow \quad \boxed{e_T = \pm\left(\frac{q_1}{Q}.e_1 + \frac{q_2}{Q}.e_2\right)} \quad (9.4)$$

Where, $\dfrac{dq_1}{Q}$ = e_1 **(error in quantity, q_1);**

$\dfrac{dq_2}{Q}$ = e_2 **(error in quantity, q_2).**

& $\dfrac{dQ}{Q}$ = e_T **(error in resultant, Q).**

ii. Difference of the Quantities:

Here, $Q = q_1 - q_2$

or, $$\frac{dQ}{Q} = \frac{d(q_1-q_2)}{Q} = \frac{dq_1}{Q} - \frac{dq_2}{Q}$$

or, $$\frac{dQ}{Q} = \frac{q_1}{Q}.\frac{dq_1}{q_1} - \frac{q_2}{Q}.\frac{dq_2}{q_2}$$

For worst possible condition, error is taken as its maximum value so above equation may be considered as:

$$\frac{dQ}{Q} = \frac{q_1}{Q}.\frac{dq_1}{q_1} + \frac{q_2}{Q}.\frac{dq_2}{q_2}$$

$$\Rightarrow \quad \boxed{e_T = \pm\left(\frac{q_1}{Q}.e_1 + \frac{q_2}{Q}.e_2\right)} \quad (9.5)$$

Both the equations (1.11.4) and (1.11.5) shows that the resulting limiting error, when quantities are added or subtracted is equal to the sum of the products formed by multiplying the individual relative errors by the ratio of such quantity to the resultant value of the quantities.

iii. Product of Quantities:

In such case; $Q = q_1 \times q_2$

Taking log on the both side,

$$\log Q = \log(q_1 \times q_2) = \log q_1 + \log q_2$$

On differentiating with respect to Q, we get

$$\frac{1}{Q}=\frac{1}{q_1}\cdot\frac{dq_1}{dQ}+\frac{1}{q_2}\cdot\frac{dq_2}{dQ}$$

or,
$$\frac{dQ}{Q}=\frac{dq_1}{q_1}+\frac{dq_2}{q_2}$$

$$\Rightarrow \quad \boxed{\boldsymbol{e_T=\pm(e_1+e_2)}} \qquad (9.6)$$

iv. Division of Quantities:

Here, $Q = q_1/q_2$

Taking log on the both sides,

$$logQ = log(q_1/q_2) = logq_1 - logq_2$$

On differentiating with respect to Q, we get

$$\frac{1}{Q}=\frac{1}{q_1}\cdot\frac{dq_1}{dQ}-\frac{1}{q_2}\cdot\frac{dq_2}{dQ}$$

or,
$$\frac{dQ}{Q}=\frac{dq_1}{q_1}-\frac{dq_2}{q_2}$$

For worst possible condition, error is taken as its maximum value so above equation may be considered as:

$$\frac{dQ}{Q}=\frac{dq_1}{q_1}+\frac{dq_2}{q_2}$$

$$\Rightarrow \quad \boxed{\boldsymbol{e_T=\pm(e_1+e_2)}} \qquad (9.7)$$

v. Power of a Factor:

Here, $Q = (q_1)^n$

Taking log on the both sides,

$$logQ = nlogq_1$$

On differentiating with respect to Q, we get

$$\frac{1}{Q}=\frac{n}{q_1}\cdot\frac{dq_1}{dQ}$$

or,
$$\frac{dQ}{Q}=n\frac{dq_1}{q_1}$$

$$\Rightarrow \quad \boxed{\boldsymbol{e_T=\pm n(e_1)}} \qquad (11.8)$$

If the product be like: $Q = (q_1)^n \times (q_2)^m$

Then
$$\boxed{\boldsymbol{e_T=\pm(ne_1+me_2)}} \qquad (11.9)$$

1.10. ERRORS IN MEASUREMENT

In any measurement system, there are several causes which are responsible for error. These errors may be classified as follows:

1. Gross errors
2. Systematic errors:
   (i) Instrumental errors;
   (ii) Environmental errors;
   (iii) Observational errors.
3. Random errors.

1. Gross Errors: The cause of these errors is human mistake in recording and reading instruments and calculating results of measurement. These errors may be of any magnitude and mathematical calculations of such errors are not easy.

e.g. Fig.11(a) and fig.11(b) are the two possible connections of pressure coil and current coil of wattmeter. The connection shown in fig.11(a) is adopted when we need to measure high voltage and low current power circuit while the connection shown in fig.11(b) is used when there is high current passes through the circuit and the applied voltage is low. If such arrangements are not considered during the measurement of power in any circuit, then the order of error in readings wattmeter will be high.

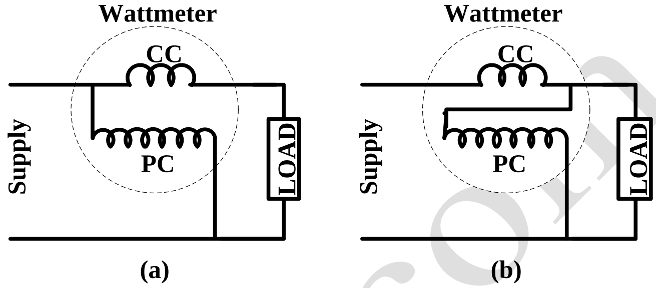

Fig.11. Wattmeter Connections

The complete elimination of these errors is probably impossible but there are some provisions to anticipate and correct those, few of the means to avoid them are:

- Be careful while taking the reading and recording the data.
- Always consider two, three or more readings of the quantity instead of just one.

2. Systematic Errors: Systematic error may be defined as, "*a constant or uniform deviation of the operation of an instrument*". These errors are remaining constant or change according to a definite law when repeated measurement of quantity is taken place. These errors are measureable and elimination of these is possible by the introduction of proper corrections.

Systematic errors may be classified as follows:

(i) Instrumental Errors: These errors occurs due to:

- Shortcomings in the instruments,
- Wrong selection or misuse of the instruments,
- Loading effects, etc.

The mechanical structure, operation or calibrations of the instrument used are the main causes of such errors. Mechanical friction, backlash and hysteresis are some examples of instrumental errors. Improper selection and poor maintenance of the instruments also are the few factors which are responsible for instrumental errors.

There are some provisions by which we can avoid these errors:

- Proper correction factor should be apply after determining the magnitude,
- Select proper measuring device for measurement purpose,
- Calibrate the measuring device periodically against a standard.

(ii) Environmental Errors: When an instrument is calibrated and assembled under some conditions but used in some other environmental conditions, these errors occurs. Change in temperature is the main cause of these errors as most of the properties of a material like-resistivity, spring effects, dimensions and others depend on temperature. Some other environmental changes such as; humidity, altitude, magnetic field of earth, gravity stray fields etc also affects the results given by instruments. There are various methods to reduce or eliminate these errors, few are given as follows:

- To use the instruments under the controlled conditions of environment in which it was originally assembled and calibrated.
- The material used should be of low temperature coefficient of resistance.
- Proper electrostatic or magnetic shields are used to provide protection against external electrostatic or magnetic fields.
- If it is required then new calibrations should be made in the new conditions.

(iii) Observational Errors: These errors are caused just because of the *carelessness* of the observer. Although the selection of instrument was proper and it was installed very carefully but if the observer is not efficient or careless about the observation, the reading will definitely deviates from actual value.

These errors may be due to the following reasons:

- Parallax- it is the very basic observational error which arises on account of pointer and scale not being in the same plane.
- Wrong scale reading and recording of wrong data.
- Inaccurate conversion of units.
- Inaccurate estimation of values of the quantities.

To avoid these errors an observer should be very careful about to reading and recording the data. Nowadays, new and modern electrical instruments are used which have digital display which provide error free outputs.

3. **Random Errors:** These errors are also known as the *accidental errors*. These errors are the result of a large number of small effects and may be a part of concern when we require a high degree of accuracy.

Random errors are of variable magnitude and do not abbey any law. To measure the quantity many times under the same conditions and calculating the average of these measurements is a method to reduce the effect of these errors.

1.11 MOVING IRON INSTRUMENTS

Moving iron instruments are one of the most accurate instrument used for both AC and DC measurement is moving iron instrument. There are two types of moving iron instrument.

Classifications: Moving iron instruments may be classified as:

(i) Attraction Type MI Instrument
(ii) Repulsion Type MI Instrument

(i) Attraction Type MI Instruments: An attraction type moving iron instrument is shown in fig.12.

Principle of operation:

In these instruments, a moving iron is used which get deflects when current (which is to be measure) passes through fixed coils. This current produces a magnetic field in fixed coils. By magnetic induction the moving iron gets magnetized. Thus the deflecting force is produced due to force of attraction. Since the moving iron is attached with the spindle, the spindle rotates and the pointer moves over the calibrated scale. The force of attraction depends on the current flowing through the coil.

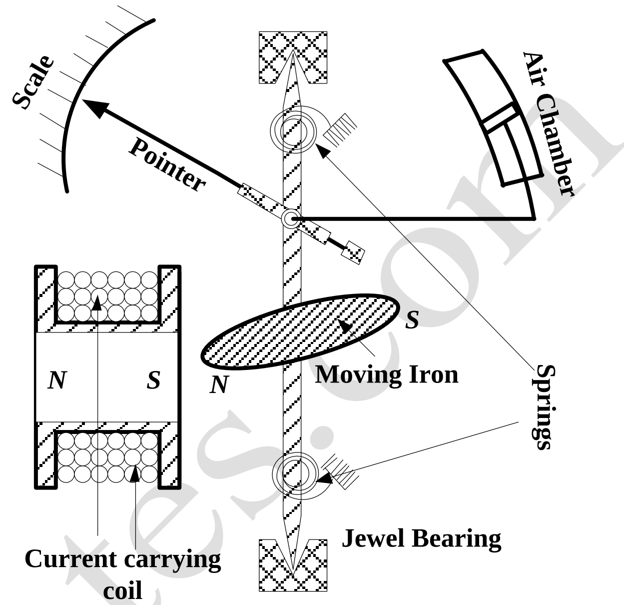

Fig.12 Attraction Type Moving Iron Insrument

Torque Equation:

Suppose, i = initial current,
L = inductance of the instrument &
θ = deflection.

Let due to the change in current 'di', the deflection change by '$d\theta$' and inductance by 'dl'.

Let the emf induced in the coil be 'e' volt, then:

$$e = \frac{d(Li)}{dt} = i\frac{dL}{dt} + L\frac{di}{dt} \quad (11.1)$$

The electrical energy supplied:

$$eidt = i^2dL + iLdi \quad (11.2)$$

During the process the stored energy changes from- $\frac{1}{2}Li^2$ to $\frac{1}{2}(L+dl)(i+di)^2$. Hence the change in energy will be:

$$\Delta E = \frac{1}{2}(L+dl)(i+di)^2 - \frac{1}{2}Li^2 \quad (11.3)$$

On neglecting second order terms of small quantities (*i.e.* di^2 & $dl.di$), above equation will be:

$$\Delta E = iLdi + \frac{1}{2}i^2dL \quad (11.4)$$

Now from the law of conservation of energy, we have:

Energy supplied = Increase in energy + mechanical work done

$$\Rightarrow \quad i^2dL + iLdi = iLdi + \frac{1}{2}i^2dL + T_d.d\theta$$

Or, $$T_d.\theta = \frac{1}{2}i^2\frac{dL}{d\theta}$$

Where, $T_d.d\theta$ = mechanical work done.

Generally the controlling torque of such instruments is provided by spring control system, for which: $T_c = K\theta$.

For steady position of pointer:

$$T_c = T_d$$

$$\Rightarrow \quad K\theta = \frac{1}{2} i^2 \frac{dL}{d\theta}$$

$$\Rightarrow \quad \text{Deflection,} \quad \boxed{\theta = \left(\frac{1}{2K}\frac{dL}{d\theta}\right).i^2} \qquad (11.5)$$

Above equation shows the square law relation, therefore the deflecting torque is unidirectional whatever may be the polarity of the current.

(ii) Repulsion type moving iron instrument:

Construction: The repulsion type instrument has a hollow fixed iron attached to it (Fig.13).

The moving iron is connected to the spindle. The pointer is also attached to the spindle in supported with jeweled bearing.

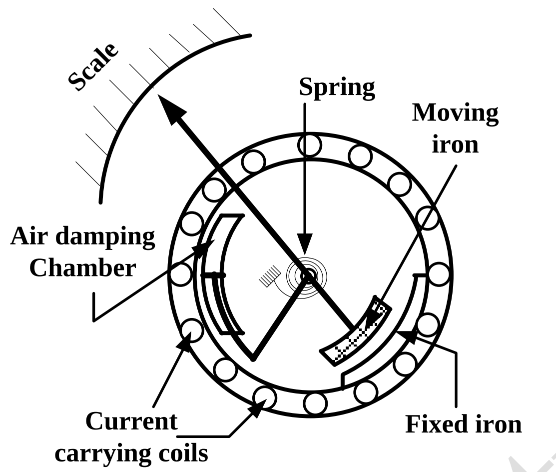

Fig.13. Repulsion Type Moving Iron Instruments

Principle of operation: When the current flows through the coil, a magnetic field is produced by it. So both fixed iron and moving iron are magnetized with the same polarity, since they are kept in the same magnetic field. Similar poles of fixed and moving iron get repelled. Thus the deflecting torque is produced due to magnetic repulsion. Since moving iron is attached to spindle, the spindle will move. So that pointer moves over the calibrated scale.

Damping: Air friction damping is used to reduce the oscillation.

Control: Spring control is used.

Advantages:

- MI can be used in AC and DC.
- It is cheap.
- Supply is given to a fixed coil, not in moving coil.
- Simple construction.
- Less friction error.

Disadvantages:

- It suffers from eddy current and hysteresis error.
- Scale is not uniform.
- It consumed more power.
- Calibration is different for AC and DC operation.

1.12 ELECTRODYNAMOMETER INSTRUMENTS

In these instruments, the operating field is produced by another coil (fixed coil) instead of permanent magnet as in the case of PMMC instruments.

Construction: The main components of these instruments are:

(1) Fixed coil: to provide magnetic field.
(2) Moving coil: it carries the current to be measured. When current passes through moving coil, an electrostatic force acts on it due to the interaction of it with magnetic field of fixed coil.
(3) Springs: Springs are used as controlling system.
(4) Air Damping chamber: To provide damping torque.
(5) Scale & Cases: for indication and safety.
(6) Shields: Shields are used to protect in instruments from stray magnetic and electrostatic fields.

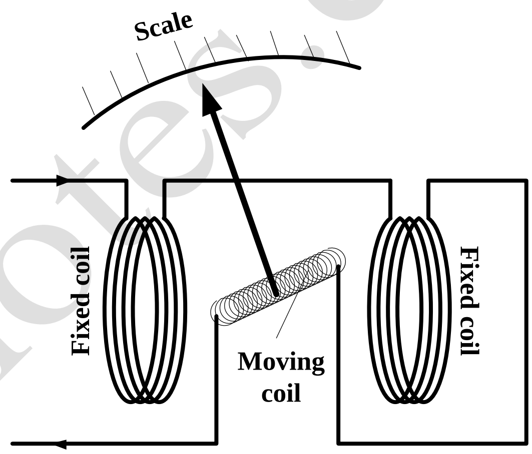

Fig.14 Electrodynamometer type Instruments

Torque Equation:

If, L_1 and L_2 are the self inductance of fixed coil and moving coil respectively & M be the mutual inductance between the coils. Then the flux linkage of the coils:

$$\psi_1 = L_1 i_1 + M i_2 \qquad (12.1)$$

$$\& \quad \psi_2 = L_2 i_2 + M i_1 \qquad (12.2)$$

The total electrical energy input:

$$E = e_1 i_1 dt + e_2 i_2 dt$$

$$= i_1 d\psi_1 + i_2 d\psi_2 \qquad \left\{\because e = \frac{d\psi}{dt}\right.$$

Substitute the values of ψ_1 and ψ_2 in above equation, we get;

$$E = i_1 d(L_1 i_1 + M i_2) + i_2 d(L_2 i_2 + M i_1)$$

$$= \left(i_1 L_1 di_1 + i_1^2 dL_1 + i_1 i_2 dM + i_1 M di_2\right) + \left(i_2 L_2 di_2 + i_2^2 dL_2 + i_1 i_2 dM + i_2 M di_1\right) \qquad (12.3)$$

Now,

$$\text{Energy stored in field} = \frac{1}{2} i_1^2 L_1 + \frac{1}{2} i_2^2 L_2 + i_1 i_2 M \qquad (12.4)$$

So,

The change in energy stored:

$$dE = d\left(\frac{1}{2} i_1^2 L_1 + \frac{1}{2} i_2^2 L_2 + i_1 i_2 M\right)$$

$$= i_1 L_1 di_1 + \frac{i_1^2}{2} dL_1 + i_2 L_2 di_2 + \frac{i_2^2}{2} dL_2 + i_1 M di_2 + i_2 M di_1 + i_1 i_2 dM \qquad (12.5)$$

From principle of conservation of energy;
Electrical energy input = change in energy stored in the field + Mechanical work done

If due to the angular deflection $d\theta$, the developed torque is T_d, then $T_d.d\theta$ will be the mechanical work done by the system.

So with the help of eq.(14.3) and eq.(14.5) the above equation may be rewrite as:

$$i_1 L_1 di_1 + i_1^2 dL_1 + 2 i_1 i_2 dM + i_1 M di_2 + i_2 L_2 di_2 + i_2^2 dL_2 + i_2 M di_1$$
$$= i_1 L_1 di_1 + \frac{i_1^2}{2} dL_1 + i_2 L_2 di_2 + \frac{i_2^2}{2} dL_2 + i_1 M di_2 + i_2 M di_1 + i_1 i_2 dM + T_d.d\theta$$

or, $$T_d.d\theta = \frac{i_1^2}{2} dL_1 + \frac{i_2^2}{2} dL_2 + i_1 i_2 dM$$

Self-inductance of the coils remains constant, *i.e.* dL_1 and dL_2 will be zero.

Hence, $$T_d.d\theta = i_1 i_2 dM$$

or, $$\boxed{T_d = i_1 i_2 \frac{dM}{d\theta}} \qquad (12.6)$$

For DC Operation: $i_1 = I_1$ and $i_2 = I_2$

So deflecting torque, $$T_d = I_1 I_2 \frac{dM}{d\theta} \qquad (12.7)$$

For spring controlled system, the controlling torque:
$$T_c = k.\theta \qquad (12.8)$$

Under steady state of pointer; $T_c = T_d$

$$\Rightarrow \quad k\theta = I_1 I_2 \frac{dM}{d\theta}$$

or, $$\boxed{\theta = I_1 I_2 \left(\frac{1}{k}\frac{dM}{d\theta}\right)} \qquad (12.9)$$

For ac operation:

For sinusoidal currents: $i_1 = I_{m1} \sin \omega t$ and $i_2 = I_{m2} \sin(\omega t - \phi)$

$\therefore$ average value of torque:

$$T_d = \frac{1}{2\pi}\int_0^{2\pi} I_{m1} \sin \omega t . I_{m2} \sin(\omega t - \phi) . \frac{dM}{d\theta} d(\omega t)$$
$$= \frac{I_{m1} I_{m2}}{2} \cos\phi . \frac{dM}{d\theta}$$

or, $$\boxed{T_d = I_1 I_2 \cos\phi . \frac{dM}{d\theta}} \qquad (12.10)$$

At equilibrium, $T_c = T_d$

or, $$k\theta = I_1 I_2 \cos\phi . \frac{dM}{d\theta}$$

$$\Rightarrow \quad \boxed{\theta = \frac{I_1 I_2}{k} \cos\phi . \frac{dM}{d\theta}} \qquad (12.11)$$

As Ammeters: Fig.15(a) shows the basic configuration when an dynamometer type instruments are used as an ammeter.

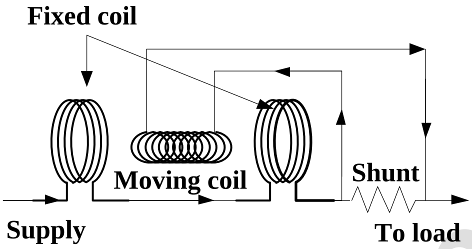

Fig.15(a) Electrodynamometer instruments as an ammeter

In such case both the moving coil current and fixed coil currents are equal to each other, *i.e.* $I_1 \approx I_2 = I$, so $\cos\phi \approx 1$, then;

$$T_d = I^2 \frac{dM}{d\theta}$$

$$\Rightarrow \quad \boxed{\theta = \frac{I^2}{k}\frac{dM}{d\theta}} \qquad (12.12)$$

As Voltmeter: The connection of an electrodynamometer type instrument, as a voltmeter is shown in fig.15(b).

In this case, $I_1 = I_2 = \frac{V}{Z}$ and $\cos\phi \approx 1$, then;

$$T_d = \frac{V^2}{Z^2}\frac{dM}{d\theta}$$

$$\Rightarrow \quad \boxed{\theta = \frac{V^2}{kZ^2}\frac{dM}{d\theta}} \qquad (12.13)$$

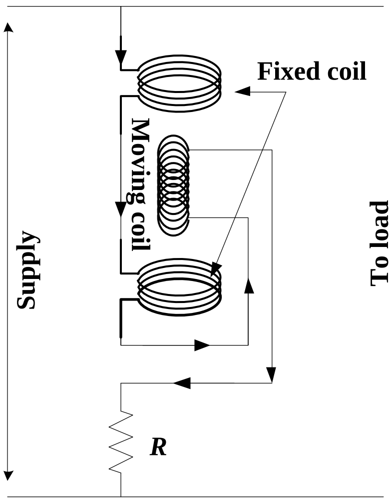

Fig.15(b) Electrodynamometer instruments as a voltmeter

Advantages:
- Useful in both ac and dc measurement.
- Free from hysteresis and eddy current losses.
- Precise and accurate instruments and same calibration for dc and ac both.

Disadvantages:
- Torque/weight ratio is small.
- Low sensitivity.
- High cost.
- Non uniform scale (in case of voltmeter and ammeter).

1.13 WATTMETER

Wattmeter is an integrating instrument, use to measure power in a circuit. It consists (refer to fig.11.):

(1) Current coil (CC) which carries the load current and
(2) Pressure coil (PC) or voltage coil carries a current proportional to, and in-phase with voltage.

1.13.1 ELECTRODYNAMOMETER WATTMETER

An electrodynamometer wattmeter is shown in fig.16.

If the coils are connected so that a value of current proportional to the load voltage flows in one coil and a value of current proportional to the load current flows in other coil, the meter may be calibrated directly in watts because indication of pointer depends upon the product of the two magnetic fields.

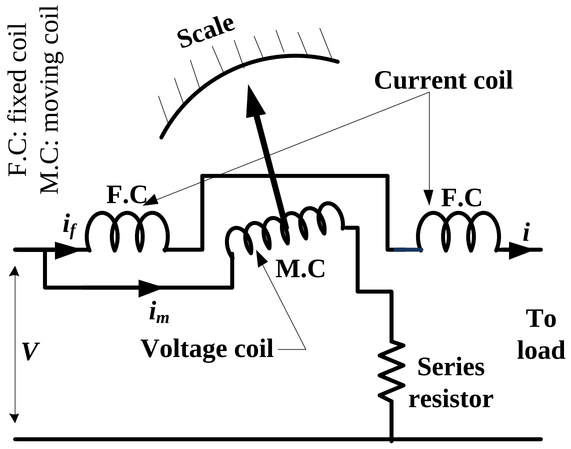

Fig.16 Dynamometer Wattmeter

For;

v : supply voltage,
i : load current,
R : equivalent resistance of moving coil,
i_f : fixed coil current = load current, I,
i_m : current through moving coil = v/R,

Deflecting torque, $T_d \propto i_f \times i_m = \dfrac{iv}{R}$ (13.1)

Now, from eq(1.14.10), the average torque in such wattmeters may be write as: $(T_d)_{avg} \propto I_m I_f \cos\phi . \dfrac{dM}{d\theta}$

$$\propto VI \cos\phi . \frac{dM}{d\theta}$$

Here, I : *r.m.s* value of load current,
V : *r.m.s* value of voltage.

For stable condition:

$$k\theta \propto (VI \cos\phi).\frac{dM}{d\theta}$$

$$\Rightarrow \quad \theta \propto (VI \cos\phi).\frac{1}{k}\frac{dM}{d\theta}$$

In above equation, $VIcos\phi$ = power of the circuit (P), so the above equation may be rewrite as:

$$\Rightarrow \quad \boxed{\theta \propto P\left(\frac{1}{k}\frac{dM}{d\theta}\right)} \qquad (13.2)$$

From above equation, it is clear that deflection is directly proportional to the power being measured and scale essentially uniform over the range in which $\dfrac{dM}{d\theta}$ is almost constant.

1.13.2 ERRORS IN WATTMETERS

(1) Error due to power loss in current coil and pressure coil: As discussed earlier, there are two possible connections of coils in a wattmeter as shown in fig.17.

In connection (a):

Power indicated by the wattmeter = Power consumed by load + Power loss in current coil

or $P = P_L + I^2R$ (13.3)

In connection (b):

Power indicated by the wattmeter = Power consumed by load + Power loss in pressure coil

or $P = P_L + I_p^2(r_p + R)$

or $\boxed{P = P_L + \dfrac{V^2}{(r_p + R)}}$ (13.4)

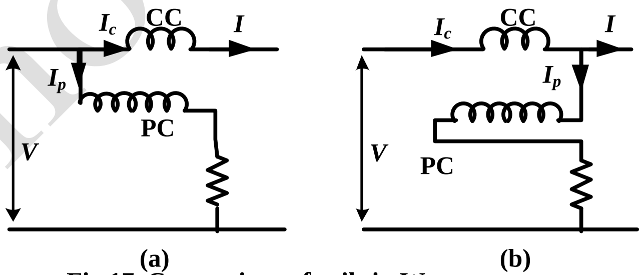

Fig.17. Connections of coils in Wattmeter

From above equations, it is clear that; for small load current, power loss in current coil is small so that connection (a) introduces a small error. On the other hand, for large load current, the power loss in the voltage coil will be small in comparison to power loss in load; hence connection (b) will be preferred.

(2) Errors due to Friction: In order to reduce friction error, the weight of moving system be reduced to the minimum and great care must be taken in the pivoting on the other hand.

(3) Errors due to Temperature: Any change in temperature changes the resistance of the pressure coil and the stiffness of the springs. To avoid these effects proper selection of material of coils and springs is very important.

(4) Errors due to Stray Fields: These instruments has relatively weak operating field and therefore, very much affected by stray magnetic and magnetic fields. To avoid these effects proper shields are provided in these instruments.

(5) Error Due to Induction of Pressure Coil:

As we know that, for proper operation of these wattmeters, current through the voltage coil should be remain in phase with voltage but due the presence of inductance of the coil the current lags behind the voltage which causes error in the circuit.

Let, V : voltage across the pressure coil,
I_p : current through the pressure coil,

Then, $$I_P = \frac{V}{\sqrt{(R+r_p)^2 + \omega^2 L_p^2}}$$

Where, R: resistance of coil connected in series with pressure coil,
r_p: Resistance of the pressure coil,
L_p: Inductance of the pressure coil.

If I_p lags behind the voltage V by an angle θ (phasor in fig.18), then; $\theta = tan^{-1}\left(\frac{\omega L_p}{r_p + R}\right)$.

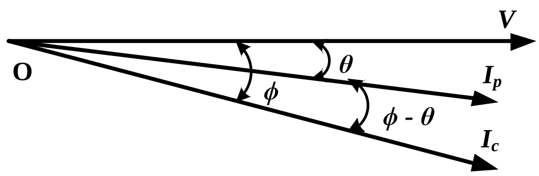

Fig.18. Phasor diagram between voltage and currents of wattmeter

From the phasor, the angle between I_p and I_c is (ϕ - θ). Now according to torque equation of electrodynamometer instruments –

Deflection $\propto I_p.I_c cos(\phi - \theta)$

or Deflection $\propto \frac{V}{Z_p}.I_c cos(\phi - \theta)$,

where; Z_p: impedance of pressure coil.

In above equation, $I_c = I$ (load current) and $Z_p = \frac{R+r_p}{cos\theta}$

So, Deflection $\propto \frac{VI}{R+r_p}\cos\theta\cos(\phi-\theta)$ (13.5)

Above equation represents the power measured by the wattmeter, *i.e.* the reading of wattmeter.

If inductance of the pressure coil is zero, then $\theta = 0$.

In this case, Deflection $\propto \frac{VI}{R+r_p}cos\,\phi$ (13.6)

Eq.(15.6) gives the true value of power.
Now,

$$\frac{\text{True power}}{\text{Reading of wattmeter}} = \frac{\frac{VI}{R+r_p}cos\,\phi}{\frac{VI}{R+r_p}cos\,\theta\,cos(\phi-\theta)}$$

Or,

$$\boxed{\textbf{True Power} = \frac{cos\,\phi}{cos\,\theta\,cos(\phi-\theta)} \times \textbf{Reading of wattmeter}} \quad (13.7)$$

The term, $\frac{cos\,\phi}{cos\,\theta\,cos(\phi-\theta)}$ is known as "*correction factor*" for lagging power factor.

Its value for leading power factor $= \frac{cos\,\phi}{cos\,\theta\,cos(\phi+\theta)}$.

Now,

Error = Reading of wattmeter – true reading

$$= \frac{VI}{R+r_p}cos\,\theta\,cos(\phi-\theta) - \frac{VI}{R+r_p}cos\,\phi$$

$$= \left(1 - \frac{cos\,\phi}{cos\,\theta\,cos(\phi-\theta)}\right)\frac{VI}{R+r_p}cos\,\phi\,cos(\phi-\theta)$$

$$= \left(1 - \frac{cos\,\phi}{cos\,\theta(cos\,\phi\,cos\,\theta + sin\,\phi\,sin\,\theta)}\right) \times \text{Actual reading}$$

Consider $cos\theta$ as unity, we get;

Error $= \left(1 - \frac{cos\,\phi}{(cos\,\phi + sin\,\phi\,sin\,\theta)}\right) \times$ Actual reading

$$= \left(\frac{cos\,\phi + sin\,\theta\,sin\,\phi - cos\,\phi}{(cos\,\phi + sin\,\phi\,sin\,\theta)}\right) \times \text{Actual reading}$$

or, $$\boxed{\textbf{Error} = \left(\frac{sin\,\theta}{(cot\,\phi + sin\,\theta)}\right) \times \textbf{Actual Reading}} \quad (13.8)$$

Above equation indicates that; for lagging power factor, instrument gives high reading and vice-versa.

(6) Error due to Capacitance of Pressure coil: The high value series resistor of pressure coil may have capacitance due to inter-turn capacitance.

If the capacitive reactance of pressure coil is equal to its inductive reactance, there will be no error due to effects since these two individual errors neutralize one another.

(7) Error due to Eddy Currents: The eddy currents sets up a magnetic field which combined with that of the current coil, produces a resultant magnetic field which is less than that of the current coil alone and also lags behind the current coil field by a small angle.

1.14 THERMOCOUPLE INSTRUMENTS

The working of these instruments are based on '***Seebeck effect***', according to which; "***when two dissimilar (properly selected) materials are joined together in the form of junctions and if these junctions are kept at different temperatures, an e.m.f is sets up between the junctions and for closed circuit, a current flows in the circuit***". The junction which kept at higher temperature is called '***hot junction***' while the junction of lower temperature is called '***cold junction***'. This system is called '***thermocouple***'.

Thermal *e.m.f.* developed in circuit:

$$E = a(T_1 - T_2) + b(T_1 - T_2)^2$$

Here, $T_1 > T_2$. Let $T_1 - T_2 = \Delta\theta$ in ^{0}C.

$$\therefore \quad E = a(\Delta\theta) + b(\Delta\theta)^2$$

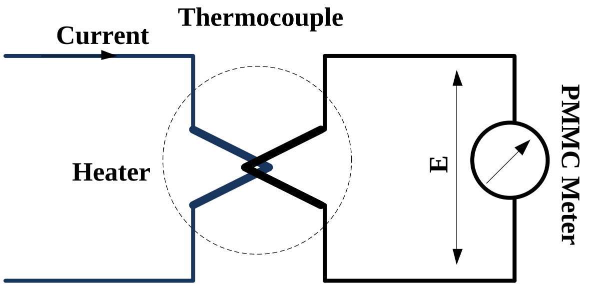

Fig.19 Circuit of a Thermocouple Instrument

The temperature rise (or difference) is proportional to the heat produced, therefore; $\Delta\theta \propto I^2R$

or, $$\Delta\theta = K_1 I^2R \qquad (14.1)$$

a, *b* are metal dependent constants. Generally *b* is very small as compare to *a*. So developed *e.m.f.* may be rewrite as:

$$E = a(\Delta\theta) \qquad (14.2)$$

This *e.m.f.* is fed to a PMMC instrument, which gives the desired deflection, proportional to the *e.m.f. E*.

i.e. Deflection: $\theta = K_2E = K_2\, a(\Delta\theta) = K_1K_2aI^2R$

or, $$\boldsymbol{\theta = KI^2} \qquad (14.3)$$

Hence the instrument shows a square law response.

Advantages:

- Suitable for high frequencies (up to 50MHz).
- Indicate *r.m.s.* value.
- There is no effect of stray magnetic field.
- Good sensitivity.
- Very useful as transfer instruments.

Disadvantages:

- Limited overload capacity.
- Low accuracy.
- Considerable power losses due to poor efficiency of the thermal conversion.

1.15 ELECTROSTATIC INSTRUMENT:

These instruments work on the force acting between the charged plates. In these instruments, one plate is fixed and the other is movable, which is controlled by a spring.

Force and Torque Equation:

(1) Linear Motion: The basic arrangement of the instrument is as shown in fig.20 (a).

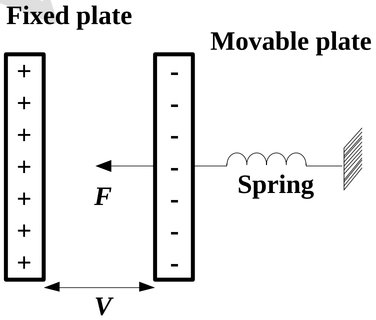

Fig.20(a) Electrostatic Instruments (Linear motion)

If, *V*: voltage applied between the plates,
C: capacitance between the plates.

Then, the energy stored to the system = $\frac{1}{2}CV^2$.

If there is a change *dV* in voltage, the force of attraction (*F*) will increased and movable plate will move by a distance *dx*.

Then the capacitive current:

$$i = \frac{dQ}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt} + V\frac{dC}{dt} \qquad (15.1)$$

The energy input is:

$$Vi.dt = V^2dC + CVdV \qquad (15.2)$$

And the change in energy:

$$= \frac{1}{2}(C + dC)(V + dV)^2 - \frac{1}{2}CV^2$$

(On neglecting the higher order terms)

Change in energy $= \frac{1}{2}V^2dC + CVdV \qquad (15.3)$

From the law of conservation of energy:

Energy input = Change in energy + mechanical work done

i.e. $V^2dC + CVdV = \frac{1}{2}V^2dC + CVdV + F.dx$

$$\Rightarrow \quad \boxed{\boldsymbol{F = \frac{1}{2}V^2\frac{dC}{dx}}} \qquad (15.4)$$

(2) Rotational Motion: If the plates are arranged as per the fig.17 (b), then during the electrostatic attraction, movable plate will rotate along its axis. In such case the expression for torque may be write with the help of eq.(4), which will be:

$$\Rightarrow \quad \boxed{\boldsymbol{T_d = \frac{1}{2}V^2\frac{dC}{d\theta}}} \qquad (15.5)$$

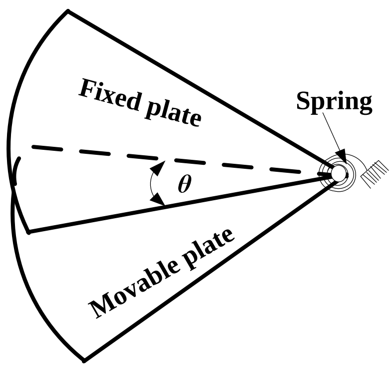

Fig.20(b) Electrostatic Instruments (Rotational motion)

For spring controlled system; $T_c = k\theta$.

Under the balanced condition of pointer; $T_c = T_d$

i.e. $$k\theta = \frac{1}{2}V^2\frac{dC}{d\theta}$$

$$\Rightarrow \quad \boxed{\boldsymbol{\theta = \frac{1}{2k}V^2\frac{dC}{d\theta}}} \qquad (15.6)$$

Advantages:

- These are transfer instruments and may be used on both ac and dc.

- Suitable for high voltages.
- Free from hysteresis and eddy current losses.
- No heating error.

Disadvantages:

- These instruments are expensive, bulky and not very robust.
- Non uniform scale.
- Since sufficient voltage is required to develop appreciable deflecting torque, hence such instruments cannot be used for measurement of low voltages.